Audubon Public School District

Algebra 2

Curriculum Guide

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Course Description

Algebra 2

The Algebra II course begins with a review of basic elementary algebra; however, stress is placed on the structure of algebra as a mathematical system and deductive reasoning. Topics used to develop real number concepts and skills are: operating with integers, solution of linear equations and inequalities, verbal problems and properties of polynomials and rational expressions. The second half of the course develops the concepts of relations and functions, emphasizing linear and quadratics, exponents, logarithms, progressions, special polynomial functions, matrices and determinants. After irrational numbers are introduced and used in these areas thus completing the real number system, the complex numbers are discussed via the imaginary numbers.

Overview / Progressions

Overview	Standards for Mathematical Content	Unit Focus	Standards for Mathematical Practice
Unit 1 Linear Functions & Systems; Quadratic Functions & Equations	● A.CED.A.1-3 ● A.REI.B.4 ● A.REI.C.6-7 ● A.REI.D.11 ● A.SSE.A.2 ● A.SSE.B.3 ● F.BF.A.1-2 ● F.BF.B.3 ● F.IF.A.3 ● F.IF.B.4-6 ● F.IF.C.7 ● F.LE.A.2 ● N.CN.A.1-3 ● N.CN.C.7 ● S.ID.B.6	● Work with linear equations examining relationships between slopes and y intercepts ● Understand concepts of a function and use function notation to represent linear functions ● Solve linear systems algebraically and graphically ● Understand Quadratic Functions and Equations ● Apply various methods of solving and identifying zeros	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments & critique the reasoning. of others. MP.4 Model with mathematics.
Unit 2 Polynomial Functions; Rational Functions	● A.APR.A.1 ● A.APR.B.2-3 ● A.APR.C.4-5 ● A.APR.D.6-7 ● A.CED.A.1-2	● Know how to combine Polynomial Functions, identify zeros, and apply transformations	

	• A.REI.A.1-2 • A.REI.D.11 • A.SSE.A.2 • F.BF.A.1 • F.BF.B.3 • F.IF.B.4,6 • F.IF.C.7,9 • N.CN.C.8-9	• Examine how exponent values, zeros, and factors affect the appearance of a graph • Define and evaluate Rational Functions • Identify the graph of a rational function • Extend operations to a rational function, solve, and graph	MP.5 Use appropriate tools strategically.. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.
Unit 3 Rational Exponents and Radical Functions; Exponential and Logarithmic Functions	• A.CED.A.1,4 • A.REI.A.1,2 • A.SSE.A.1-2 • A.SSE.B.3-4 • F.BF.A.1-2 • F.BF.B.3-5 • F.IF.A.3 • F.IF.B.4-6 • F.IF.C.7-9 • F.LE.A.2,4 • F.LE.B.5 • N.RN.A.1-2 • S.ID.B.6	• Use properties of rational exponents and radicals • Analyze and solve radical functions • Identify relationship between Exponential and Logarithmic Functions and use inverse operations to solve	

		• Work with geometric sequences and series	
Unit 4 Trigonometric Functions; Statistics; and Probability	• A.CED.A.3 • A.REI.C.8-9 • F.BF.B.3 • F.IF.B.4,6 • F.IF.C.9 • F.TF.A.1-3 • F.TF.B.5 • F.TF.C.8 • G.SRT.D.10-11 • N.Q.A.2 • S.CP.A.1-5 • S.CP.B.6-9 • S.IC.A.1-2 • S.IC.B.3-6 • S.ID.A.1-2,4 • S.MD.A.1-4 • S.MD.B.5,7,9	• Determine probability when order is involved and identify what changes when items are repeated • Use right triangle trigonometry to extend to the unit circle • Identify and work with basic trigonometry functions • Apply algebra to the trigonometric functions to find key features such as	

		amplitude, period, and shift.	

Subject: Algebra 2	**Grade: 10-12**	**Unit: 1**	**1st Marking Period**
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	
● F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. ● F.IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	MP.3 Construct viable arguments & critique the reasoning. of others. MP.4 Model with mathematics. MP.6 Attend to precision.	Concept(s): ● No new concepts introduced Students are able to: ● Write the domain and range of functions using set builder and interval notations ● Identify key features of a graph of a function ● Calculate and interpret the average rate of change of a function over a specified interval Learning Goal 1: ● Identify and interpret key features of the graph of a function including domain, range, intercepts, and areas where the graph is increasing and decreasing.	
● F.BF.B.3. Identify the effect on the graph of replacing f(x) by	MP.4 Model with	Concept(s):	

f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. ● F.IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	mathematics. MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	● Transformations of functions by changing values of *a*, *b*, *h*, or *k* Students are able to: ● Graph a transformed function by identifying the effect on the graph of replacing *f(x)* by *f(x)+k*, *kf(x)*, *f(kx)*, and *f(x+k)* for specific values of *k*. ● Write an equation of a transformed function ● Relate the domain of a function to its graph and the real-world situation it describes Learning Goal 2: ● Write equations for linear and quadratic functions by understanding how changing the values of *a*, *b*, *h*, and *k* affect the key features of the graph of a function.

● F.IF.C.7 o F.IF.C.7.B. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ● S.ID.B.6 o S.ID.B.6.A. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.	MP.3 Construct viable arguments & critique the reasoning. of others. MP.6 Attend to precision. MP.7 Look for and make use of structure.	Concept(s): ● Model situations where there are different rules for different parts of the function using piecewise-functions Students are able to: ● Create and graph piecewise-defined functions, including absolute value functions and step functions ● Create and use a piecewise-defined function from real-world data ● Write a piecewise-defined rule from a graph Learning Goal 3: ● Understand that an absolute value function is a piecewise-defined function. Create and graph piecewise-defined functions including absolute value functions and step functions.

● F.BF.A.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, and use them to model situations, and translate between the two forms. ● F.IF.A.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	MP.3 Construct viable arguments & critique the reasoning. of others. MP.4 Model with mathematics. MP.7 Look for and make use of structure.	Concept(s): ● Definite an arithmetic sequence Students are able to: ● Identify the common difference in an arithmetic sequence ● Write arithmetic sequences both recursively and with an explicit formula ● Construct arithmetic sequences, given a graph, a description of a relationship, or two input-output pairs Learning Goal 4: ● Write the general rule for an arithmetic sequence recursively as a piecewise-defined function and then translate from a recursive formula to an explicit formula.
● A.CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	MP.3 Construct viable arguments & critique the reasoning. of others. MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	Concept(s): ● Solve equations and inequalities by graphing Students are able to: ● Use graphs, tables, and graphing technology to find or approximate solutions to equations and inequalities ● Find approximate solutions to equations and inequalities by setting each expression equal to *y* and graphing Learning Goal 5: ● Solve an equation or inequality by setting each expression equal to *y*,

		graphing the equations, and interpreting the intersection of the graphs.
● A.REI.C.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. ● A.CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	Concept(s): ● Solution to a system is a set of ordered pairs satisfying all equations and/or inequalities in system Students are able to: ● Solve linear systems graphically and algebraically ● Identify regions that solve systems of inequalities Learning Goal 6: ● Solve systems of linear equations and inequalities using graphing or elimination. Write systems of linear equations as a matrix.
● A.CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ● F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal	MP.1 Make sense of problems and persevere in solving them. MP.3 Construct viable arguments & critique the reasoning. of others. MP.7 Look for and make use of structure.	Concept(s): ● All quadratic functions are transformations of the parent function ● Vertex form highlights key features and shows how the graph of the parent function can be transformed Students are able to: ● Create quadratic functions in vertex form to represent relationships between variables as shown in their graphs. ● Graph functions on coordinate axes using their key features ● Interpret key features of the graph of a quadratic function

description of the relationship. Key features include: intercept; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.		Learning Goal 7: ● Create and graph quadratic functions in vertex form by identifying and interpreting the key features.
● A.CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ● F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercept; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.	MP.3 Construct viable arguments & critique the reasoning. of others. MP.4 Model with mathematics. MP.7 Look for and make use of structure.	Concept(s): ● Quadratic function in vertex form can be rewritten in standard form to highlight different features Students are able to: ● Create quadratic functions written in standard form ● Identify key features of quadratic functions and graph a quadratic function written in standard form Learning Goal 8: ● Rewrite quadratic functions in standard form. Graph a quadratic function written in standard form by identifying its key features.

● A.APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. ● A.SSE.B.3 ○ A.SSE.B.3.A. Factor a quadratic expression to reveal the zeros of the function it defines.	MP.1 Make sense of problems and persevere in solving them. MP.3 Construct viable arguments & critique the reasoning. of others. MP.7 Look for and make use of structure.	Concept(s): ● Factored form of a quadratic function is used to find zeros of the function Students are able to: • Write a quadratic equation in factored form and use it to identify the zeros of the function it defines • Determine the intervals over which a quadratic function is positive or negative Learning Goal 9: ● Factor a quadratic expression to find the zeros of a quadratic function. Use the Zero Product Property to solve quadratic equations by factoring.
● N.CN.A.1. Know there is a complex number i such that i 2 = –1, and every complex number has the form a + bi with a and b real. ● N.CN.A.2. Use the relation i 2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	MP.4 Model with mathematics. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	Concept(s): ● Complex numbers contain both real and imaginary parts ● Four basic operations can be applied to complex numbers Students are able to: • Add, subtract, and multiply complex numbers using the properties of operations and the relation $i^2=-1$. • Use complex numbers to represent numbers that are not on the real number line. Learning Goal 10: ● Solve quadratic equations with complex solutions and understand that a complex number includes both the real and imaginary parts. Use

		properties of operations to add, subtract, and multiply complex numbers.
● A.REI.B.4 ○ A.REI.B.4.A. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x − p) 2 = q that has the same solutions. Derive the quadratic formula from this form. ● A.SSE.B.3 ○ A.SSE.B.3.B. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	MP.3 Construct viable arguments & critique the reasoning. of others. MP.6 Attend to precision. MP.7 Look for and make use of structure.	Concept(s): ● Quadratic equations can be solved by completing the square to transform the equation to an equivalent equation Students are able to: ● Transform a quadratic equation into the form $(x-p)^2=q$ by completing the square. ● Complete the square to reveal the minimum or maximum value of a quadratic expression. Learning Goal 11: ● Solve quadratic equations by completing the square. Find the minimum or maximum value of a quadratic expression.

● A.REI.B.4 ○ A.REI.B.4.B. Solve quadratic equations by inspection (e.g., for x 2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. ● N.CN.C.7. Solve quadratic equations with real coefficients that have complex solutions.	MP.3 Construct viable arguments & critique the reasoning. of others. MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	Concept(s): ● Quadratic formula can be used to solve any quadratic equation Students are able to: ● Use the Quadratic Formula to solve quadratic equations that have complex solutions Learning Goal 12: ● Use completing the square to derive the Quadratic Formula and then use the Quadratic Formula to solve quadratic equations with real and complex roots.
● A.REI.C.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. ● A.REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the	MP.3 Construct viable arguments & critique the reasoning. of others. MP.4 Model with mathematics. MP.7 Look for and make use of structure.	Concept(s): ● Linear-quadratic systems consist of linear and quadratic equations Students are able to: ● Use algebra to solve a linear-quadratic system ● Solve a linear-quadratic system using graphing and explain why the points of intersection are the solutions Learning Goal 13: ● Solve linear-quadratic systems in two variables algebraically and

solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.		graphically. Explain why the points where the graphs intersect are the solutions to the system.

Formative Assessments	**Summative Assessments**
• Independent, guided, and group practice/activities • Teacher observation • Technology result data (desmos, quizlet, quizizz, kahoot, IXL, etc.) • Checks for Understanding • Pre-Assessment	• Mid-chapter and chapter standard aligned assessments (tests and quizzes) • Post Unit Assessment
Suggested Primary Resources	**Suggested Supplemental Resources**
• enVision Algebra 2 • TI 84 Graphing Calculator (https://parcctrng.testnav.com/client/index.html#login?username=17MTA1PTOE01010200&password=PCPRACTICE) • TI 89 Graphing Calculator (Honors) • Desmos Graphing Calculator (www.desmos.com)	• Desmos Classroom Activities • Khan Academy Tutorials • Kuta Software LLC • IXL Math • Edpuzzle • Quizlet, Quizizz, etc
Cross-Curricular Connections & 21st Century Skills	
• Science, Technology, Engineering, and Mathematics (STEM) Literacy	

- Critical Thinking and Problem Solving
- Communication and Collaboration
- Life and Career Skills

Essential Questions	Enduring Understanding
● What are the relationships between slope, y-intercept and linear equations? ● What are functions and their rules? ● What are the properties and rules of exponents? ● How can we use patterns on numbers to solve problems? ● How can number patterns be classified? ● What are quadratic equations and how are they solved? ● What do quadratic functions look like when they are graphed? ● What are complex numbers & imaginary numbers?	● Algebra can be used to model real world problems. Practical problems can be interpreted, represented, and solved using equations. ● Functions are special types of relations, which can be evaluated, graphed and combined. ● Multiplying exponents is not like multiplying regular numbers; there are special rules that must be followed. ● Sequences and series arise from practical situations. The study of sequences and series is an application of investigation of patterns. ● There are several ways to solve polynomial equations. Complex numbers are a superset of real numbers.

Differentiation & Real World Connections

504	● preferential seating ● extended time on tests and assignments ● reduced homework or classwork ● verbal, visual, or technology aids	● modified textbooks or audio-video materials ● behavior management support ● adjusted class schedules or grading ● verbal testing
Enrichment	● Utilize collaborative media tools ● Provide differentiated feedback ● Opportunities for reflection ● Opportunities for self-evaluation	● Encourage student voice and input ● Model close reading ● Distinguish long term and short term goals

IEP	• Utilize “skeleton notes” where some required information is already filled in for the student • Provide access to a variety of tools for responses • Provide opportunities to build familiarity and to practice with multiple media tools • Graphic organizers	• Leveled text and activities that adapt as students build skills • Provide multiple means of action and expression • Consider learning styles and interests • Provide differentiated mentors
ELLs	• Pre-teach new vocabulary and meaning of symbols • Embed glossaries or definitions • Provide translations • Connect new vocabulary to background knowledge	• Provide flash cards • Incorporate as many learning senses as possible • Portray structure, relationships, and associations through concept webs • Graphic organizers
At-risk	• Purposeful seating • Counselor involvement • Parent involvement	• Contracts • Alternate assessments • Hands-on learning
21st Century Skills		
• Creativity • Innovation • Critical Thinking		• Problem Solving • Communication • Collaboration
Integrating Technology		

• Chromebooks • Internet research • Online programs	• Virtual collaboration and projects • Presentations using presentation hardware and software
Career education	
• Weekly Discussions: The value of mastering multiple languages in the workforce.	• Equity Discussions: People who benefit from knowing multiple languages.

Subject: Algebra 2	**Grade: 10-12**	**Unit: 2**	**2nd Marking Period**
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	
• F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercept; intervals where the function is increasing, decreasing, positive, or negative; relative maximums	MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	Concept(s): • Graph and identify key features of a polynomial function • Use leading coefficient and degree of polynomial function to predict end behavior of its graph Students are able to: • Graph polynomial functions and show the key features of the graph • Predict the end behavior of polynomial functions by interpreting the leading coefficients and degrees • Sketch graphs showing key features, given a verbal description Learning Goal 1:	

and minimums; symmetries; end behavior; and periodicity. ● F.IF.C.7 o F.IF.C.7.c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.		● A polynomial function is a function whose rule is either a monomial or a sum of monomials. Key features of a polynomial function can be used to sketch a graph of the function.
● A.APR.A.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. ● F.IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments & critique the reasoning. of others.	Concept(s): ● Add, subtract, and multiply polynomials ● Compare properties of polynomials represented in different ways Students are able to: ● Add, subtract, and multiply polynomials and understand that polynomials are closed under these operations ● Compare a polynomial function represented algebraically with one represented graphically Learning Goal 2: ● As with real numbers, properties of operations can be used to add, subtract, and multiply polynomials. Polynomial functions can be used to represent and compare real-world situations.

● A.APR.C.4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.* ● A.APR.C.5. Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	Concept(s): ● Know and apply Binomial Theorem ● Use polynomial identities to multiply and factor Students are able to: ● Prove polynomial identities and use them to multiply and factor polynomials ● Expand binomials using the Binomial Theorem and coefficients determined by Pascal's Triangle Learning Goal 3: ● Polynomial identities and the Binomial Theorem are helpful tools for efficiently rewriting expressions and describing mathematical relationships
● A.APR.B.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. ● A.APR.D.6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of	MP.2 Reason abstractly and quantitatively. MP.6 Attend to precision.	Concept(s): ● Divide polynomials using long and synthetic division ● Use Remainder Theorem to evaluate polynomials ● Use Factor Theorem to identify factors Students are able to: ● Divide polynomial expressions using long division ● Use synthetic division to rewrite rational expressions Learning Goal 4:

$r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.		● Polynomial expressions can be divided by linear factors using long or synthetic division. The Remainder Theorem is used to determine the remainder of a polynomial division problem.
● A.APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. ● F.IF.C.7 ○ F.IF.C.7.C. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	MP.7 Look for and make use of structure MP.8 Look for and express regularity in repeated reasoning	Concept(s): ● Use factoring or synthetic division to identify zeros of a polynomial function ● Use zeros to sketch a graph of a function defined by the polynomial Students are able to: ● Identify the zeros of a function by factoring or using synthetic division ● Use the zeros of a polynomial function to sketch its graph Learning Goal 5: ● The zeros of a polynomial function can be determined using factoring or synthetic division. The zeros of a function can be used to sketch its graph.
● F.BF.B.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using	MP.1 Make sense of problems and persevere in solving them	Concept(s): ● Graph and transform the reciprocal function ● Identify how the values of a, h, and k affect the key features of a transformation of the reciprocal function Students are able to: ● Use inverse variation to write and graph the reciprocal function

technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.		• Identify the effect of transformations on the graph of the reciprocal function and define the effects of *h* and *k* on the function $□(□) = \frac{1}{□-h} + □$ Learning Goal 6: • The reciprocal function is used to model inverse variation, which is a proportional relationship between two variables such that when one variable increases, the other decreases.
• F.IF.C.7 ○ F.IF.C.7.D. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. • A.APR.D.6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	Concept(s): • Graphing rational functions • Identify key features of the graphs of rational functions, such as asymptotes Students are able to: • Graph rational functions by identifying asymptotes and end behavior • Rewrite simple rational expressions in different forms using long division Learning Goal 7: • A rational function is any function $□(□) = \frac{□(□)}{□(□)}$ where *P(x)* and *Q(x)* are polynomial functions. The domain of a rational function is all real numbers except any *x*-values for which *Q(x)* equals to zero. The graph of a rational function has one or more asymptotes, which guide the end behavior of the graph.
• A.APR.D.6. Rewrite simple rational expressions in	MP.6 Attend to precision.	Concept(s):

different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	MP.7 Look for and make use of structure.	• Multiply and divide rational expressions Students are able to: • Use the structure of rational expressions to rewrite simple rational expressions in different forms • Understand that rational expressions form a system of analogous to the system of rational numbers and use that understanding to multiply and divide rational expressions Learning Goal 8: • Rational expressions form a system similar to the system of rational numbers and can be multiplied and divided by applying the properties of operations as they apply to rational expressions
• A.SSE.A.2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* • A.APR.D.7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression;	MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	Concept(s): • Add and subtract rational expressions Students are able to: • Understand that rational expressions form a system analogous to the system of rational numbers and use that understanding to add and subtract rational expressions Learning Goal 9: • The properties of operations used to add and subtract rational numbers can be applied to adding and subtracting rational expressions

add, subtract, multiply, and divide rational expressions.		
● A.REI.A.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. ● A.REI.A.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	MP.1 Make sense of problems and persevere in solving them. MP.7 Look for and make use of structure.	Concept(s): ● Solve rational equations ● Identify extraneous solutions Students are able to: ● Solve rational equations in one variable ● Identify extraneous solutions to rational equations and give examples of how they arise Learning Goal 10: ● Rational equations contain a rational expression and can be solved by multiplying each side of the equation by a common denominator to eliminate the fractions. Any solution that is excluded from the domain of the original equation is extraneous.

Formative Assessments	**Summative Assessments**
● Independent, guided, and group practice/activities ● Teacher observation ● Technology result data (desmos, quizlet, quizizz, kahoot, IXL, etc.) ● Checks for Understanding ● Pre-Assessment	● Mid-chapter and chapter standard aligned assessments (tests and quizzes) ● Post Unit Assessment
Suggested Primary Resources	**Suggested Supplemental Resources**
● enVision Algebra 2	● Desmos Classroom Activities ● Khan Academy Tutorials

• TI 84 Graphing Calculator (https://parcctrng.testnav.com/client/index.html#login?username=17MTA1PTOE01010200&password=PCPRACTICE) • TI 89 Graphing Calculator (Honors) • Desmos Graphing Calculator (www.desmos.com)	• Kuta Software LLC • IXL Math • Edpuzzle • Quizlet, Quizizz, etc
Cross-Curricular Connections & 21st Century Skills	
• Science, Technology, Engineering, and Mathematics (STEM) Literacy • Critical Thinking and Problem Solving • Communication and Collaboration • Life and Career Skills	
Essential Questions	**Enduring Understanding**
• How can a polynomial inequality be solved? • How can real world data be used to generate a polynomial model? • What does it mean to be an odd or even function? • How do the elements of a polynomial equation determine its general shape? • How can the solutions or zeros of a polynomial be found? • How do the zeros of a polynomial relate to its graph? • How are operations extended to rational functions? • How are rational expressions simplified? • How are rational equations solved? • How are rational equations graphed?	• Odd and even functions have graphs that are symmetric with respect to the origin or y-axis. Zeros of a polynomial can be found by factoring or by graphing the polynomial. • Solve rational expressions. Computational skills applicable to numerical fractions also apply to rational expressions involving variables.

Differentiation & Real World Connections

504	● preferential seating ● extended time on tests and assignments ● reduced homework or classwork ● verbal, visual, or technology aids	● modified textbooks or audio-video materials ● behavior management support ● adjusted class schedules or grading ● verbal testing
Enrichment	• Utilize collaborative media tools • Provide differentiated feedback • Opportunities for reflection • Opportunities for self-evaluation	• Encourage student voice and input • Model close reading • Distinguish long term and short term goals
IEP	• Utilize “skeleton notes” where some required information is already filled in for the student • Provide access to a variety of tools for responses • Provide opportunities to build familiarity and to practice with multiple media tools • Graphic organizers	• Leveled text and activities that adapt as students build skills • Provide multiple means of action and expression • Consider learning styles and interests • Provide differentiated mentors
ELLs	• Pre-teach new vocabulary and meaning of symbols • Embed glossaries or definitions • Provide translations • Connect new vocabulary to background knowledge	• Provide flash cards • Incorporate as many learning senses as possible • Portray structure, relationships, and associations through concept webs • Graphic organizers
At-risk	• Purposeful seating • Counselor involvement • Parent involvement	• Contracts • Alternate assessments • Hands-on learning
21st Century Skills		

• Creativity • Innovation • Critical Thinking	• Problem Solving • Communication • Collaboration
Integrating Technology	
• Chromebooks • Internet research • Online programs	• Virtual collaboration and projects • Presentations using presentation hardware and software
Career education	
• Weekly Discussions: The value of mastering multiple languages in the workforce.	• Equity Discussions: People who benefit from knowing multiple languages.

Subject: Algebra 2	**Grade: 10-12**	**Unit: 3**	**3rd Marking Period**
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	
• N.RN.A.1. Explain how the definition of the meaning of rational exponents follows from extending the properties	MP.1 Make sense of problems and persevere in solving them.	Concept(s): • Find *n*th roots of a number and use them to rewrite expressions • Solve equations involving rational exponents	

of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* ● N.RN.A.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	MP.5 Use appropriate tools strategically.	Students are able to: ● Use properties of exponents to rewrite expressions involving radicals in terms of rational exponents ● Find all real *n*th roots of a number ● Evaluate expressions with rational exponents ● Use *n*th roots to solve equations by rewriting expressions using the properties of exponents Learning Goal 1: ● Rational exponents and radicals represent the number of roots a polynomial has. The roots of a polynomial are used to simplify expressions and solve equations.
● A.SSE.A.1. Interpret expressions that represent a quantity in terms of its context. ● A.SSE.A.2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of*	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	Concept(s): ● Use properties of radicals and exponents to rewrite radical expressions Students are able to: ● Use the properties of exponents and radicals to identify ways to rewrite radical expressions ● Interpret radical expressions that represent a quantity in terms of its context

squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.		Learning Goal 2: ● The properties of integer exponents can be applied to terms with rational exponents, as well as to radicals. The properties of exponents and radicals can be used to rewrite radical expressions. When rewriting radical expressions, like radicals, which have the same index, can be added and subtracted.
● F.IF.C.7 ● F.IF.C.7.B Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ● F.BF.B.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	Concept(s): ● Graph and transform radical functions Students are able to: ● Graph radical functions, including square root and cube root functions ● Identify the effect of transformations on the key features of the graphs of radical functions Learning Goal 3: ● The function $□(□) = □\sqrt[□]{□ - h} + □$ represents the transformation of the parent radical function $□(□) = \sqrt[□]{□}$, where a stretches or compresses the graph vertically, h translates the graph horizontally, and k translates the graph vertically.

● A.REI.A.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. ● A.REI.A.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	MP.3 Construct viable arguments & critique the reasoning of others. MP.7 Look for and make use of structure.	Concept(s): ● Solve radical equations, identifying extraneous solutions Students are able to: ● Solve radical equations in one variable ● Explain how extraneous solutions may arise when solving radical equations ● Solve radical inequalities and apply the solution within a real-world context Learning Goal 4: ● Solving equations that include radicals or rational exponents is similar to solving rational equations, except that rather than eliminating the rational expression through multiplication, the radical expression is eliminated by raising both sides of the equation to the power equal to the index of the radical
● F.IF.C.7 ○ F.IF.C.7.E. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	MP.4 Model with mathematics. MP.7 Look for and make use of structure.	Concept(s): ● Graph exponential functions ● Interpret key features of exponential functions by examining tables and graphs Students are able to: ● Interpret key features of exponential functions represented by tables, graphs, and equations

● F.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.		● Graph transformations of exponential functions showing intercepts and end behavior ● Model quantities that increase or decrease by a fixed percent each time period using exponential functions Learning Goal 5: ● The rate of exponential growth or decay is the ratio between two consecutive output values in an exponential function
● F.IF.C.8 ● F.IF.C.8.B. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay. ● F.LE.B.5. Interpret the parameters in a linear or exponential function in terms of a context. ● S.ID.B.6	MP.1 Make sense of problems and persevere in solving them. MP.4 Model with mathematics.	Concept(s): ● Analyze real-world situations using exponential models Students are able to: ● Rewrite exponential functions to identify rates ● Interpret the parameters of an exponential function within the context of compound interest problems ● Construct exponential models given two points or by using regression Learning Goal 6: ● Exponential models are useful for representing situations in which the rate increases by the same percent for each period of time and for interpreting problems that involve compound interest. Exponential regression can be used to generate exponential models for real-world contexts.

● S.ID.B.6.A. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.		
● F.BF.B.4 ○ F.BF.B.4.A. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2x³ or f(x) = (x+1)/(x-1) for x ≠ 1.* ● F.BF.B.5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. F.LE.A.4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	Concept(s): ● Learn about the relationship between exponents and logarithms and use this relationship to solve problems ● Evaluate common and natural logarithms and solve equations involving logarithms Students are able to: ● Understand the inverse relationship between exponents and logarithms ● Use logarithms to solve exponential models ● Evaluate logarithms using technology Learning Goal 7: ● A logarithmic function is the inverse of an exponential function. Logarithms are found by determining the exponent that must be applied to a base to yield a given result.

e; evaluate the logarithm using technology.		
● F.BF.B.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	MP.4 Model with mathematics. MP.7 Look for and make use of structure.	Concept(s): ● Find key features of logarithmic graphs, interpret logarithm models, and their inverses Students are able to: ● Graph logarithmic functions and interpret their key features ● Write and interpret the inverses of exponential and logarithmic functions Learning Goal 8: ● The inverse relationship between exponential and logarithmic functions reveals key features of the graphs of both functions. Logarithmic functions can be used to model real-world situations.
● A.SSE.A.2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	Concept(s): ● Use inverse relationship between exponential and logarithmic functions to prove Properties of Logarithms ● Simplify and expand expressions using Properties of Logarithms Students are able to: ● Use Properties of Logarithms to rewrite logarithmic expressions ● Use Change of Base Formula to evaluate logarithmic expressions and solve equations

		Learning Goal 9: ● Properties of Logarithms can be used to rewrite logarithmic expressions and to evaluate logarithms by changing the base.
● A.SSE.A.2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* ● A.CED.A.1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	Concept(s): ● Create and solve exponential and logarithmic equations and construct arguments to justify the solution methods Students are able to: ● Use logarithms to express the solutions to exponential models ● Solve exponential and logarithmic equations Learning Goal 10: ● Some exponential equations can be solved by rewriting both sides with a common base. For others, rewriting the equation using logarithms and applying properties of logarithms, is a more efficient method.
● F.LE.A.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	MP.2 Reason abstractly and quantitatively.	Concept(s): ● Write geometric sequences recursively and with an explicit formula and translate between the two forms ● Find the sum of a geometric series Students are able to:

		• Construct a geometric sequence given a graph, table, or description of a relationship • Translate between geometric sequences written in recursive and explicit forms • Use the formula for the sum of a finite geometric series to solve problems Learning Goal 11: • A geometric sequence is a sequence of numbers in which terms are related to the previous term by a common ratio, *r*. A geometric series is the sum of a certain number of terms in a geometric sequence.

Formative Assessments	**Summative Assessments**
• Independent, guided, and group practice/activities • Teacher observation • Technology result data (desmos, quizlet, quizizz, kahoot, IXL, etc.) • Checks for Understanding • Pre-Assessment	• Mid-chapter and chapter standard aligned assessments (tests and quizzes) • Post Unit Assessment
Suggested Primary Resources	**Suggested Supplemental Resources**
• enVision Algebra 2 • TI 84 Graphing Calculator (https://parcctrng.testnav.com/client/index.html#login?username=17MTA1PTOE01010200&password=PCPRACTICE) • TI 89 Graphing Calculator (Honors) • Desmos Graphing Calculator (www.desmos.com)	• Desmos Classroom Activities • Khan Academy Tutorials • Kuta Software LLC • IXL Math • Edpuzzle • Quizlet, Quizizz, etc

Cross-Curricular Connections & 21st Century Skills	
● Science, Technology, Engineering, and Mathematics (STEM) Literacy ● Critical Thinking and Problem Solving ● Communication and Collaboration ● Life and Career Skills	
Essential Questions	**Enduring Understanding**
● What are radicals? How can a radical function be identified from its graph? ● How can a radical expression be simplified and combined? ● How can a radical equation be solved? ● How can operations be extended to radical expressions and equations? ● What is the relationship between logarithms and exponential equations? ● How can they be identified from their graphs? ● How can an exponential equation be constructed to solve a real world problem? ● How can operations be extended over exponential, logarithms, natural log and natural base? ● How can equations involving logarithms, natural log and natural base be solved?	● Radicals are the opposite of exponents. Only radicals with a common radicand and index can be added or subtracted. Radical equations can be solved by using the graphing method or by finding the inverse. ● Logarithms and exponents are inverse notations. Logs are used to solve problems involving unknown exponent values and are still relied upon heavily in business, science, and financial formulas.

Differentiation & Real World Connections		
504	● preferential seating ● extended time on tests and assignments ● reduced homework or classwork ● verbal, visual, or technology aids	● modified textbooks or audio-video materials ● behavior management support ● adjusted class schedules or grading ● verbal testing

Enrichment	• Utilize collaborative media tools • Provide differentiated feedback • Opportunities for reflection • Opportunities for self-evaluation	• Encourage student voice and input • Model close reading • Distinguish long term and short term goals
IEP	• Utilize "skeleton notes" where some required information is already filled in for the student • Provide access to a variety of tools for responses • Provide opportunities to build familiarity and to practice with multiple media tools • Graphic organizers	• Leveled text and activities that adapt as students build skills • Provide multiple means of action and expression • Consider learning styles and interests • Provide differentiated mentors
ELLs	• Pre-teach new vocabulary and meaning of symbols • Embed glossaries or definitions • Provide translations • Connect new vocabulary to background knowledge	• Provide flash cards • Incorporate as many learning senses as possible • Portray structure, relationships, and associations through concept webs • Graphic organizers
At-risk	• Purposeful seating • Counselor involvement • Parent involvement	• Contracts • Alternate assessments • Hands-on learning
21st Century Skills		
• Creativity • Innovation • Critical Thinking		• Problem Solving • Communication • Collaboration

Integrating Technology	
● Chromebooks ● Internet research ● Online programs	● Virtual collaboration and projects ● Presentations using presentation hardware and software
Career education	
● Weekly Discussions: The value of mastering multiple languages in the workforce.	● Equity Discussions: People who benefit from knowing multiple languages.

Subject: Algebra 2	**Grade: 10-12**	**Unit: 4**	**4th Marking Period**
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	
● F.TF.A.3. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of	MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	Concept(s): ● Use a trigonometric ratio and the Pythagorean Theorem to find another ratio for the same angle or a missing side length of a right triangle Students are able to: ● Use special triangles to determine trigonometric ratios geometrically ● Use trigonometric functions and the Pythagorean Theorem to find missing side lengths	

their values for x, where x is any real number.		● Identify and explain trigonometric identities Learning Goal 1: For any right triangle, the ratios of the sides are always the same for any given angle □. These ratios define the six basic trigonometric functions: sine, cosine, secant, cosecant, tangent, and cotangent.
● F.TF.B.6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. ● F.TF.B.7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	Concept(s): ● Use unit circle to evaluate trigonometric functions Students are able to: ● Define and evaluate inverse trigonometric functions ● Solve trigonometric equations using inverse functions Learning Goal 2: ● An inverse trigonometric function maps each value in the range of the original function to only one value in the domain of the original function.

● G.SRT.D.10. Prove the Laws of Sines and Cosines and use them to solve problems. ● G.SRT.D.11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	Concept(s): ● Use the Law of Sines and the Law of Cosines to solve problems and to find unknown measurements of non-right triangles Students are able to: ● Derive the Law of Sines and Law of Cosines ● Use the Law of Sines and the Law of Cosines to find unknown angles and sides of non-right triangles Learning Goal 3: ● The Law of Sines or Law of Cosines can be used to find unknown measurements of a non-right triangle
● F.TF.A.4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. ● F.TF.C.9. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments & critique the reasoning of others.	Concept(s): ● Use trigonometric identities and sum and difference formulas to verify and apply relationships between trigonometric functions and solve problems Students are able to: ● Verify trigonometric identities using the unit circle ● Use trigonometric identities to rewrite expressions ● Prove sum and difference formulas for sine, cosine, and tangent, and use them to solve real-world problems Learning Goal 4:

		● A trigonometric identity is an equation that is true for all values of the variable for which both sides of the equation are defined.
● N.Q.A.2. Define appropriate quantities for the purpose of descriptive modeling. ● S.IC.A.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	MP.6 Attend to precision. MP.8 Look for and express regularity in repeated reasoning.	Concept(s): ● Understand and use vocabulary related to statistical questions and variables for the purpose of descriptive modeling Students are able to: ● Define and recognize a statistical question ● Define and identify the type of statistical variable that is represented by a question or the data represented on a graph ● Distinguish between quantities such as population/sample and parameter/statistic for the purpose of descriptive modeling Learning Goal 5: ● A statistical question is a question that can be answered by collecting many pieces of information, or data. The data can be categorical (qualitative) or statistical (quantitative). The data are measured by parameters, which describe the population, and statistics, which describe a sample of the population.
● S.IC.A.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	MP.1 Make sense of problems and persevere in solving them.	Concept(s): ● Choose best type of study to answer a given statistical question ● Choose a reasonable sample for a statistical study Students are able to: ● Identify experiments, sample surveys, and observational studies

● S.IC.B.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ● S.IC.B.6. Evaluate reports based on data.	MP.3 Construct viable arguments & critique the reasoning of others.	● Recognize bias in sampling methods ● Identify a sampling method that provides a random sample from a population Learning Goal 6: ● There are three types of statistical studies: experiments, sample surveys, and observational studies. The way in which samples are chosen for a study affects how well they represent the population. To avoid bias, samples should be random.
● S.ID.A.1. Represent data with plots on the real number line (dot plots, histograms, and box plots). ● S.ID.A.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	MP.2 Reason abstractly and quantitatively. MP.6 Attend to precision.	Concept(s): ● Use statistics such as mean, median, quartiles, and standard deviation to describe and compare data sets representing a sample of the population Students are able to: ● Find measures of center and spread, such as median, mean, interquartile range, and standard deviation ● Compare data sets using statistical measures that are appropriate for the distribution of the data Learning Goal 7: ● A data distribution can be normal, skewed left, or skewed right.

• S.ID.A.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. • S.IC.B.6. Evaluate reports based on data	MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	Concept(s): • Fit a normal distribution to data and use it to understand where a data value falls Students are able to: • Fit a normal distribution to data • Compare and contrast data using *z*-scores • Use technology to calculate the area under standard normal distribution curve Learning Goal 8: • The normal distribution is used to explain where data values fall within a population. The standard normal distribution allows for a comparison of values across different population distributions.
• S.CP.A.2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments & critique the reasoning of others.	Concept(s): • Learn to identify events as mutually exclusive or independent • Use a sum of probabilities to find the probability of the union of mutually exclusive events • Use a product of probabilities to find the probability of the intersection of independent events Students are able to:

		• Explain independence of events in everyday language and everyday situations • Determine the probability of the union of two events (*A* or *B*) and the intersection of two independent events (*A* and *B*) Learning Goal 9: • Two events that cannot both occur are mutually exclusive. Two events are independent if the occurrence of one does not affect the probability of the other. The probability that two independent events both occur is the product of their probabilities.
• S.CP.A.3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	MP.1 Make sense of problems and persevere in solving them. MP.7 Look for and make use of structure.	Concept(s): • Calculate the conditional probability of *A* given *B* by dividing *P*(*A* and *B*) by *P*(*B*) • Use conditional probability to test for independence of events Students are able to: • Understand the conditional probability of *A* given *B* as the fraction of outcomes in *B* that also belong to *A* • Interpret independence of events Learning Goal 10: The conditional probability that event *A* will occur, given that another event *B* has occurred, is written as *P*(*A*|*B*). Two events are independent if and only if *P*(*A*|*B*) = *P*(*A*) and *P*(*B*|*A*) = *P*(*B*)

● S.CP.B.9. Use permutations and combinations to compute probabilities of compound events and solve problems.	MP.3 Construct viable arguments & critique the reasoning of others. MP.7 Look for and make use of structure.	Concept(s): ● Use Fundamental Counting Principle ● Develop and apply formulas for the number of permutations and combinations of a set of items ● Use permutations and combinations to determine the number of outcomes in a situation to calculate probability Students are able to: ● Calculate the number of permutations and combination in mathematical and real-world context ● Use permutations and combinations to compute probabilities of compound events and solve problems Learning Goal 11: ● A permutation is an arrangement of items in which the order of the items matters, while a combination is an arrangement in which order does not matter

Formative Assessments	**Summative Assessments**

• Independent, guided, and group practice/activities • Teacher observation • Technology result data (desmos, quizlet, quizizz, kahoot, IXL, etc.) • Checks for Understanding • Pre-Assessment	• Mid-chapter and chapter standard aligned assessments (tests and quizzes) • Post Unit Assessment
Suggested Primary Resources	**Suggested Supplemental Resources**
• enVision Algebra 2 • TI 84 Graphing Calculator (https://parcctrng.testnav.com/client/index.html#login?username=17MTA1PTOE01010200&password=PCPRACTICE) • TI 89 Graphing Calculator (Honors) • Desmos Graphing Calculator (www.desmos.com)	• Desmos Classroom Activities • Khan Academy Tutorials • Kuta Software LLC • IXL Math • Edpuzzle • Quizlet, Quizizz, etc
Cross-Curricular Connections & 21st Century Skills	
• Science, Technology, Engineering, and Mathematics (STEM) Literacy • Critical Thinking and Problem Solving • Communication and Collaboration • Life and Career Skills	
Essential Questions	**Enduring Understanding**
• What is right triangle trigonometry? • How can right triangle definitions be extended to apply to circular functions? • What are the three basic trigonometric functions? • How is algebra used in trigonometry? • How can we use these functions to solve right triangles? • What graphic organizer best represents different types of data? • How can scatter plots be used to find patterns? • How can the equation of best fit for data be found? • How can graphic organizers be used to make predictions? • How do you find the probability when order is involved?	• Trigonometry can be used not just to solve triangles, but to model waves. The important properties of waves, including amplitude, period, and shift, can be found by applying algebra to the trigonometric function • Data and scatter plots may indicate patterns that can be modeled with an algebraic equation. Graphing calculators can be used to collect, organize, picture, and create an algebraic model of the data. Data that fit linear, quadratic, exponential, and logarithmic models arise from practical situations.

• When is order important? • What changes when items are arranged in a circle? • What changes when items are repeated?	• The desired results can help choose the best graphic organizer for a given set of data. Correlations and lines of best fit can be used to find patterns in scatterplots and to make predictions.

Differentiation & Real World Connections		
504	● preferential seating ● extended time on tests and assignments ● reduced homework or classwork ● verbal, visual, or technology aids	● modified textbooks or audio-video materials ● behavior management support ● adjusted class schedules or grading ● verbal testing
Enrichment	• Utilize collaborative media tools • Provide differentiated feedback • Opportunities for reflection • Opportunities for self-evaluation	• Encourage student voice and input • Model close reading • Distinguish long term and short term goals
IEP	• Utilize “skeleton notes” where some required information is already filled in for the student • Provide access to a variety of tools for responses • Provide opportunities to build familiarity and to practice with multiple media tools • Graphic organizers	• Leveled text and activities that adapt as students build skills • Provide multiple means of action and expression • Consider learning styles and interests • Provide differentiated mentors

ELLs	• Pre-teach new vocabulary and meaning of symbols • Embed glossaries or definitions • Provide translations • Connect new vocabulary to background knowledge	• Provide flash cards • Incorporate as many learning senses as possible • Portray structure, relationships, and associations through concept webs • Graphic organizers
At-risk	• Purposeful seating • Counselor involvement • Parent involvement	• Contracts • Alternate assessments • Hands-on learning
21st Century Skills		
• Creativity • Innovation • Critical Thinking		• Problem Solving • Communication • Collaboration
Integrating Technology		
• Chromebooks • Internet research • Online programs		• Virtual collaboration and projects • Presentations using presentation hardware and software
Career education		
• Weekly Discussions: The value of mastering multiple languages in the workforce.		• Equity Discussions: People who benefit from knowing multiple languages.

Appendix A

Audubon Public Schools
Engaging Students ~ Fostering Achievement ~ Cultivating 21st Century Global Skills
Written By: Ron Latham, Patricia Martel
Course Title: Algebra II Unit Name: Arithmetic, Linear Functions, and Geometric Sequences.
Grade Level: 9-12
Approved: August, 2014
Approved June 2017

Content Statements and Rationale: In this unit, students will connect their prior study of algebraic patterns, linear functions, and geometric sequences. Students will explore the basic characteristics of arithmetic and geometric sequences and series.	**NJSLS:** A.REI.11 A.SSE.4 F.IF.1-3, 5 F.BF.1-2 F.LE.2 S.ID.8
Overarching Essential Questions: What are the relationships between slope, y-intercept and linear equations? What are functions and their rules? What are the properties and rules of exponents? How can we use patterns on numbers to solve problems? How can number patterns be classified?	**Overarching Enduring Understandings:** Algebra can be used to model real world problems. Practical problems can be interpreted, represented, and solved using equations. Functions are special types of relations, which can be evaluated, graphed and combined. Multiplying exponents is not like multiplying regular numbers; there are special rules that must be followed. Sequences and series arise from practical situations. The study of sequences and series is an application of investigation of patterns.

Unit Essential Questions:	**Unit Enduring Understandings:**
How can you determine from a graph, scatter plot, equation or table if a relationship is linear? What methods can be used to solve a system of linear Equations? How can you determine the best method of solution? How can we use a mean slope to write an equation for real world data? What is a function? How do we define is domain and range? What are the rules of combining functions? How are functions substituted? How does a function relate to its inverse? How is an inverse of a function found graphically? Algebraically? What is $\sum$? What is the difference between a series and a sequence? What makes a series or sequence arithmetic? How can we find the next and nth term? How can the sum of the arithmetic sequence be found? What makes a series or sequence geometric? How can we find the next and nth term? How can the sum of the geometric sequence be found? Of the series?	There are relationships between slope, y-intercept and linear equations and changing each of these elements changes the appearance of the graph. Functions describe the relationship between two variables. Graphs of functions that are inverses of each other are reflections across the line $y = x$. The composition of a function and its inverse is the identity function. Functions arise from practical situations. If (a, b) is an element of a function, then (b, a) is an element of the inverse of the function. $\sum$ is the summation function? The difference between a sequence and a series is a series is infinite while a sequence is not. An arithmetic sequence or series can be identified by its applied pattern of addition or subtraction. Sums can be found by applying appropriate formulas.
Benchmarks: Algebra 2 Holt/ McDougal- Assessment Book: Ch 1 Assessment Ch 2 Assessments Prentice Hall- Algebra 2 Progress Monitoring Assessments- Benchmark Tests.	**Unit Student Learning Overview:** Students will be able to use linear equations to solve direct variation and proportion equations, use graphs, scatter plots, regression and tables to determine linear relationships, find slopes and intercepts, write, graph and use linear equations and inequalities to predict outcomes, use systems of linear equations to solve problems and model real life situations, use basic algebraic rules to simplify and solve expressions, equations, and combination of functions.

	Students will also be able to investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems.

Key Terms (Essential Vocabulary):
Combination of Functions- adding, subtracting, multiplying or dividing two or more functions
Composition of Functions- substituting one function inside another function
Direct Variation- two variable values changing at the same rate with respect to each other
Domain- the list or description of all possible x values that exist as solutions to a function
Intercept- the point where a line or curve touches or crosses an axis
Linear- being in a line
Slope- the change in y value divided by the change in x value
System of equations- two or more equations with common variables
Arithmetic- having a standard pattern of addition or subtraction
Geometric- having a standard pattern of multiplying or dividing

Resources:
Algebra 2 (ISBN 0-03-066054-8)
TI 83 or 84 Graphing Calculator
TI Exploration Activities – see TI web site for up to date plans and worksheets
Worksheets and warm up from www.kuta.com
Videos from www.brightstorm.com and www.khanacademy.com
Concord Consortium https://concord.org/
Desmos www.desmos.com
Three Acts https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTQEsfqouLWNNC6Z4/edit#gid=0

Suggested Activities for Inclusion in Lesson Planning

Interdisciplinary Connections are identified with and I, followed by the related content area(s):

Identify the domain, range, zeros, and inverse of a function presented algebraically or graphically.

Students will apply the concept of function to sets of data points to determine if the set is a function or just a relation. Then they will graph the sets of data to see the impact of being a function or relation on the graph. Students will write a set of rules to define if a graph is a relation or a function and then use these rules to determine if the graph of an equation, using the graphing calculator, is a relation or a function.

After studying the concepts of domain and range, students will graph a variety of equations using their graphing calculators to determine the domain and range of each, paying special attention to functions where there are limits on either to recognize restricted/discontinuous domains and ranges.

Introduce the concept of inverse or opposite of functions and its definition of switching x and y. After practicing the algebraic manipulation, students will use their graphing calculators to see the impact of the opposite functions on a graph.

Find the composition of two functions.

Apply the concept of input and output to function notation. Develop the concept of coordinate pairs in function notation and how they can be found on the graphing calculator. Students will find this information for a scripted set of scenarios then create scenarios to swap with a peer to solve each other's problems.

At the college prep level, extra time could be spent on domain in this unit.

Collect data for an employees pay based on amount of hours worked, including what happens to the employees pay after when they start to work overtime hours. Investigate using the internet to find salaries for student selected jobs and careers. Calculate the pay based on week or biweekly pay. Write equations to model accumulated salary over time as both linear and step functions including the impact of overtime pay.

Setup functions that represent different types of discounts, a set amount off price and a percentage off of price. Use the composition of the functions to determine which discount taken first will give a lower sales price.

Calculate the value of a maximized ROTH investment annually for a “working” lifetime.

Distinguish between a sequence and a series.

Recognize patterns in a sequence.

Students will work with a peer to generate the set of the next 10 terms of a sequence. Determine which type of sequence it forms, arithmetic, geometric or neither. Then each pair will create two recursive routines of each type to swap with a neighboring group.

Distinguish between arithmetic and geometric sequences.

Use and interpret the notations å, n, nth term, and an.

Write the first n terms in an arithmetic or geometric sequence.

Given the formula, find an (the nth term) for an arithmetic or a geometric sequence.

Given formulas, find the sum, Sn, of the first n terms of an arithmetic or geometric series, including infinite series.

Modifications for Special Education Students, ELLs and Gifted Students: **Special Needs** – Students with IEPs will be placed in classes with additional instructional support, and the material will be delivered in a co-teaching model. Students with 504s will receive the support those documents dictate. **ELL** – Language support, as needed. Utilization of experience and information, as applicable. Opportunities for students to write or communicate in their native language, as availability of translation allows. Depending on the level of acquisition, opportunities to write instead of speaking; and opportunities to practice speaking. **Gifted Learners** – Deeper investigations of content; lead roles during collaborative group assignments. **Mainstream Learners** – Formative assessments to gauge understanding and learning; participation in lesson examples.	**Suggested Timeline:** Seeing Structure in Expressions: **1 week** Interpreting and Building Functions: **1 week** Linear, Quadratic, and Exponential Models: **1 week** Sequences and Series: **1-2 weeks**

Audubon Public Schools
Engaging Students ~ Fostering Achievement ~ Cultivating 21st Century Global Skills
Written By: Ron Latham, Patricia Martel
Course Title: Algebra II Unit Name: Quadratic Relations and Equations
Grade Level: 9-12

Content Statements and Rationale: In this unit, students will focus on quadratic equations, as they utilize the same concepts for analysis as linear equations, roots, zeros, end behaviors, and inverse.	**NJSLS:** N.Q.2 N.CN.1-2, 7 A.REI.4,7 F.BF.1,3 G.GPE.2 S.ID.6
Overarching Essential Questions: What are quadratic equations and how are they solved? What do quadratic functions look like when they are graphed? What are complex numbers & imaginary numbers?	**Overarching Enduring Understandings:** There are several ways to solve polynomial equations. Complex numbers are a superset of real numbers.
Unit Essential Questions: What does it mean to be quadratic? How can a quadratic expression be factored? What are the methods to solving a quadratic equation? How do we select the best method? What are complex numbers? How can they be manipulated? What are imaginary numbers? How can they be manipulated? How can a polynomial inequality be solved? How can real world data be used to generate a polynomial model? What does it mean to be an odd or even function?	**Unit Enduring Understandings:** Complex numbers have square roots that cannot be simplified. Imaginary numbers come from numbers that have negative square roots. Odd and even functions have graphs that are symmetric with respect to the origin or y-axis. Zeros of a polynomial can be found by factoring or by graphing the polynomial.

How do the elements of a polynomial equation determine its general shape? How can the solutions or zeros of a polynomial be found? How do the zeros of a polynomial relate to its graph?	
Benchmarks: Algebra 2 Holt/ McDougal- Assessment Book: Ch 4 Assessment Prentice Hall- Algebra 2 Progress Monitoring Assessments- Benchmark Tests.	**Unit Student Learning Overview:** Students will be able to find solutions or zeros of quadratic functions using a variety of methods including: graphing, substitution, factoring, division, completing the square, and quadratic formula.
Key Terms (Essential Vocabulary): Binomial- having two terms Complex number- a binomial having both real and imaginary terms Constant- a number Factoring- dividing or reversing the distribution Imaginary number- the square root of a negative number Monomial- having one term Quadratic- having the highest power of 2 as an exponent Quadratic Formula – a method of finding solutions to quadratic equations Trinomial- having three terms	
Resources: Algebra 2 (ISBN 0-03-066054-8) TI 83 or 84 Graphing Calculator TI Exploration Activities – see TI web site for up to date plans and worksheets Worksheets and warm up from www.kuta.com Videos from www.brightstorm.com and www.khanacademy.com Desmos www.desmos.com Three Acts https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTQEsfqouLWNNC6Z4/edit#gid=0	

Suggested Activities for Inclusion in Lesson Planning

Interdisciplinary Connections are identified with and I, followed by the related content area(s):

Setup a quadratic equation that will solve to the width of a patio on two sides of house when given the dimensions of the house and the total square feet of material.

"A gardener is putting a wire fence along the edge of his garden to keep animals from eating his plants. If he has 20 meters of fence, what is the largest rectangular area he can enclose?

- To find the area of a rectangle, what two quantities do you need? Choose on to be your variable and write the other in terms of this variable.
- How can a graph help you solve this problem?
- What quadratic functions represents the area of the garden?

Use a scatter plot that represents that relationship of the speed of a car and its stopping distance. Find a quadratic model that can find a cars stopping distance depending on any speed.

Special attention needs to be made to review of factoring and extended to include factoring patterns and factoring of quadronomials.

Identify and graph quadratic equations using pencil and paper and calculator methods.

Factor quadratic expressions using patterns and methods.

Solve quadratic equations using factoring, quadratic formula, graphing and completing the square. After completing lessons on each method of solution, give students a selection of problems and have students, in small groups, determine which method is best suited to each problem, provide their reasoning, and use that method to find the solution.

Students will use regression to find a series of equations that pass through a predetermined set of targets or points to map the path that a spacecraft will pass through to collect data.

Students will play the Factor This! Game, in teams of four constructing their own challenge problems for the opposing team to solve meeting selected criteria.

Students will play Dominoes game: select a tile and construct an equation that has those two solutions and pass it to an opponent who will reverse the process back to the solutions.

Use the discriminant to test for solutions to quadratics. Graph a series of quadratic equations using the calculator and evaluate the discriminant to determine how the value of the discriminant impacts the solutions to the equations.

Simplify complex numbers using algebraic rules.

Graph quadratic inequalities and systems.

Apply distribution process to binomial and trinomial distribution.

Recognize that the square root of –1 is represented as i.

Define and identify a complex number.

Apply the definition of *i* to simplify square roots of negative numbers. Simplify powers of *i*. Add, subtract, and multiply complex numbers.	
Modifications for Special Education Students, ELLs and Gifted Students: **Special Needs** – Students with IEPs will be placed in classes with additional instructional support, and the material will be delivered in a co-teaching model. Students with 504s will receive the support those documents dictate. **ELL –** Language support, as needed. Utilization of experience and information, as applicable. Opportunities for students to write or communicate in their native language, as availability of translation allows. Depending on the level of acquisition, opportunities to write instead of speaking; and opportunities to practice speaking. **Gifted Learners** – Deeper investigations of content; lead roles during collaborative group assignments. **Mainstream Learners** – Formative assessments to gauge understanding and learning; participation in lesson examples.	**Suggested Timeline:** Quantities and the Complex Number System: **1-2 weeks** Solving quadratic equations, Solving Systems of Equations, Reasoning with Equations & Inequalities: **1-2 weeks** Building Functions and Expressing Geometric Properties with Equations: **1-2 weeks**

Written By: Ron Latham, Patricia Martel
Course Title: Algebra II Unit Name: Polynomial Functions and Equations
Grade Level: 9-12

Content Statements and Rationale: In this unit, we will expand the concepts of Domain, Range, inverses, roots, end behaviors, and zeros from quadratics to all polynomials.	**NJSLS:** N.CN.9 A.SSE.2 A.APR.2-4 A.REI.11 F.IF.4, 6-7, 9 F.BF.1,3
Overarching Essential Questions: How do exponent value, zeros, and factors affect the appearance of a graph?	**Overarching Enduring Understandings:** Odd functions begin and end in opposite directions. Even functions begin and end in the same direction. Zeros are the x intercepts of an equation and can be used to find factors and write a polynomial equation.
Unit Essential Questions: How can a polynomial inequality be solved? How can real world data be used to generate a polynomial model? What does it mean to be an odd or even function? How do the elements of a polynomial equation determine its general shape? How can the solutions or zeros of a polynomial be found? How do the zeros of a polynomial relate to its graph?	**Unit Enduring Understandings:** Odd and even functions have graphs that are symmetric with respect to the origin or y-axis. Zeros of a polynomial can be found by factoring or by graphing the polynomial.
Benchmarks: Algebra 2 Holt/ McDougal- Assessment Book: Ch 5 Assessment	**Unit Student Learning Overview:** Students will be able to find solutions or zeros of polynomial functions using a variety of methods including: graphing, substitution, factoring,

Prentice Hall- Algebra 2 Progress Monitoring Assessments- Benchmark Tests.	division, synthetic division and substitution, and quadratic formula.

Key Terms (Essential Vocabulary):
Constant- a number
Cubic- having the highest power of 3 as an exponent
Quadratic Formula – a method of finding solutions to quadratic equations
Quadronomial- having 4 terms
Quartic- having the highest power of 4 as an exponent
Quintic- having the highest power of 5 as an exponent
Synthetic Division- a method used to find solutions to polynomial equations
Trinomial- having three terms

Resources:
Algebra 2 (ISBN 0-03-066054-8)
TI 83 or 84 Graphing Calculator
TI Exploration Activities – see TI web site for up to date plans and worksheets
Worksheets and warm up from www.kuta.com
Videos from www.brightstorm.com and www.khanacademy.com
Desmos www.desmos.com
Three Acts
https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTQEsfqouLWNNC6Z4/edit#gid=0

Suggested Activities for Inclusion in Lesson Planning
Interdisciplinary Connections are identified with and I, followed by the related content area(s):
Apply the Rational Root theorem to find roots to test with synthetic process.
Add, subtract, plot and multiply complex numbers.
Investigate the shape and behavior of linear, quadratic, and cubic functions. Behaviors will include intercepts, number of turning points, and end behavior.
Using the general shape of the graph of a function, identify the family of graphs to which a particular graph belongs. Characteristics of a graph may include the x- and y-intercepts, number and location of turning points, and end behaviors.
Plot complex numbers on the complex number plane and use the resultant graph to answer a question.

Modifications for Special Education Students, ELLs and Gifted Students:	**Suggested Timeline:**
Special Needs – Students with IEPs will be placed in classes with additional instructional support, and the material will be delivered in a co-teaching model. Students with 504s will receive the support those documents dictate. **ELL** – Language support, as needed. Utilization of experience and information, as applicable. Opportunities for students to write or communicate in their native language, as availability of translation allows. Depending on the level of acquisition, opportunities to write instead of speaking; and opportunities to practice speaking. **Gifted Learners** – Deeper investigations of content; lead roles during collaborative group assignments.	Seeing Structure in Polynomial Expressions: **1 week** Arithmetic with Polynomials and Rational Expressions: **1-2 weeks** Interpreting and Building Polynomial Functions: **1 week**

Mainstream Learners – Formative assessments to gauge understanding and learning; participation in lesson examples.	

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Written By: Ron Latham, Patricia Martel
Course Title: Algebra II Unit Name: Rational Functions and Equations
Grade Level: 9-12

Content Statements and Rationale: In this unit, students will study the general characteristics and behavior of rational functions and apply their knowledge to transforming functions to create and understand graphs of rational functions.	**NJSLS:** A.SSE.2 A.APR.6 A.CED.1 A.REI.1-2, 11 F.IF.7 F.BF.1,4
Overarching Essential Questions: What is a rational function and what does its graph look like?	**Overarching Enduring Understandings:** Rational functions can be represented as fractional exponents and follow the same rules as regular exponents.

Unit Essential Questions: How are operations extended to rational functions? How are rational expressions simplified? How are rational equations solved? How are rational equations graphed?	**Unit Enduring Understandings:** Solve rational expressions. Computational skills applicable to numerical fractions also apply to rational expressions involving variables.
Benchmarks: Algebra 2 Holt/ McDougal- Assessment Book: Ch 6 Assessment Prentice Hall- Algebra 2 Progress Monitoring Assessments- Benchmark Tests.	**Unit Student Learning Overview:** Students will be able to apply skills from polynomials to identify, graph, simplify, and solve rational expressions. Students will also be able to add, subtract, multiply, divide, and simplify rational expressions, including complex fractions.
Key Terms (Essential Vocabulary): Asymptote- an invisible line that the function does not cross but approaches Hole- having a gap in value due to a common factor in the numerator and denominator Horizontal- following the horizon line Rational- having a terminating or repeating value Rationalize- a process that eliminates imaginary or irrational numbers from the denominator of a rational function Vertical – being upright	
Resources: Algebra 2 (ISBN 0-03-066054-8) TI 83 or 84 Graphing Calculator TI Exploration Activities – see TI web site for up to date plans and worksheets Worksheets and warm up from www.kuta.com Videos from www.brightstorm.com and www.khanacademy.com Desmos www.desmos.com	

Three Acts
https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTQEsfqouLWNNC6Z4/edit#gid=0

Suggested Activities for Inclusion in Lesson Planning
Interdisciplinary Connections are identified with and I, followed by the related content area(s):
Setup a combined variation to determine that speed a bicycle is traveling based on the number of teeth the gears have and their number of revolutions per minute.
Setup a rational expression that represents the distance to and from a location and different speeds for each direction. Use the expression to solve for the drivers average speed, which will be slightly lower than the average of the two speeds because more time will be spent on the road at a slower speed.
Add, subtract, multiply, and divide rational expressions whose denominators are monomials or polynomial
expressions in completely factored form.
Solve inverse variation problems and check the answers using the decoding activity.
Graphionary: provide student groups with a series of graphed rational expressions and student groups must create correct rational expression equations for a subset of four forming a line on the grid. The fastest correct set of answers receive extra credit points.
Simplify a rational expression with common monomial or binomial factors.
Recognize a complex fraction, and simplify it as a quotient or product of simple fractions.
Identify from a graph the solutions to an equation containing rational expressions.
Solve equations containing rational expressions with monomial denominators algebraically and graphically.
Check possible solutions to an equation containing rational expressions, using a graphing calculator.

Modifications for Special Education Students, ELLs and Gifted Students:	**Suggested Timeline:**
Special Needs – Students with IEPs will be placed in classes with additional instructional support, and the material will be delivered in a co-teaching	Seeing Structure in Rational Expressions: **1 week** Arithmetic with Polynomials and Rational Expressions, Creating Equations, and Reasoning with Equations and Inequalities: **1-2 weeks**

model. Students with 504s will receive the support those documents dictate. **ELL –** Language support, as needed. Utilization of experience and information, as applicable. Opportunities for students to write or communicate in their native language, as availability of translation allows. Depending on the level of acquisition, opportunities to write instead of speaking; and opportunities to practice speaking. **Gifted Learners** – Deeper investigations of content; lead roles during collaborative group assignments. **Mainstream Learners** – Formative assessments to gauge understanding and learning; participation in lesson examples.	Interpreting and Building Rational Functions: **1 week**

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Written By: Ron Latham, Patricia Martel
Course Title: Algebra II Unit Name: Radical Functions and Equations
Grade Level: 9-12

Content Statements and Rationale:	**NJSLS:**
In this unit, students will study transformations on the parent square root function to model data and formulate equations arising from the square root functions. Students will also explore solutions for	N.Q.2 N.RN.1-2 A.REI.1-2 F.IF.4

these equations using tables and graphs, and learn how the inverse relationship between the square root and quadratic functions relate in solving these radical equations.	F.BF.4
Overarching Essential Questions: What are radicals and how can they be simplified? What do graphs of radical functions look like?	**Overarching Enduring Understandings:** Radical expressions can be written and simplified using rational exponents. Graphs of radical functions look like curves, are not symmetric and can be transformed using predictive indicators.
Unit Essential Questions: What are radicals? How can a radical function be identified from its graph? How can a radical expression be simplified and combined? How can a radical equation be solved? How can operations be extended to radical expressions and equations?	**Unit Enduring Understandings:** Radicals are the opposite of exponents. Only radicals with a common radicand and index can be added or subtracted. Radical equations can be solved by using the graphing method or by finding the inverse.
Benchmarks: Algebra 2 Holt/ McDougal- Assessment Book: Ch 7 Assessment Prentice Hall- Algebra 2 Progress Monitoring Assessments- Benchmark Tests.	**Unit Student Learning Overview:** Students will be able to add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents. Students will also be able to write radical expressions as expressions containing rational exponents and vice versa.
Key Terms (Essential Vocabulary): Radical- the frame in which the expression is contained Radicand- the contents of the radical	

Resources:
Algebra 2 (ISBN 0-03-066054-8)
TI 83 or 84 Graphing Calculator
TI Exploration Activities – see TI web site for up to date plans and worksheets
Worksheets and warm up from www.kuta.com
Videos from www.brightstorm.com and www.khanacademy.com
Desmos www.desmos.com
Three Acts
https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTQEsfqouLWNNC6Z4/edit#gid=0

Suggested Activities for Inclusion in Lesson Planning
Interdisciplinary Connections are identified with and I, followed by the related content area(s):
Use radical equations to solve exponential problems involving income. Use radical equations to find out the interest rate necessary to meet savings targets over various periods of time. Investigate current and past interest rates to determine the viability of the interest rate solution.
Simplify radical expressions containing positive rational numbers and variables.
Convert from radical notation to exponential notation, and vice versa.
Add and subtract radical expressions with like radicands.
Multiply and divide radical expressions not requiring rationalizing the denominators.
Solve equations containing a radical expression algebraically and graphically. The equation will contain a linear expression under the radical, and all terms outside the radical will be constants.
Identify from a graph the solutions to an equation containing radical expressions.
Check possible solutions to an equation containing radical expressions, using a graphing calculator.

Modifications for Special Education Students, ELLs and Gifted Students:	**Suggested Timeline:**
Special Needs – Students with IEPs will be placed in classes with additional instructional support, and the material will be delivered in a co-teaching	Extend the properties of Exponents to Rational Exponents, Solving Radical Equations in One Variable: **1 week** Interpreting and Building Radical Functions, Finding Inverse Functions: **1 week**

model. Students with 504s will receive the support those documents dictate. **ELL –** Language support, as needed. Utilization of experience and information, as applicable. Opportunities for students to write or communicate in their native language, as availability of translation allows. Depending on the level of acquisition, opportunities to write instead of speaking; and opportunities to practice speaking. **Gifted Learners** – Deeper investigations of content; lead roles during collaborative group assignments. **Mainstream Learners** – Formative assessments to gauge understanding and learning; participation in lesson examples.	

Content Statements and Rationale: In this unit, students will strengthen their understanding of the inverse relationship while making connections between exponential and logarithmic functions.	**NJSLS:** N.Q.2 N.SSE.3 A.CED.1 A.REI.11 F.IF.4, 6-9 F.BF.1, 3, 5 F.LE.2, 4-5 S.ID.6
Overarching Essential Questions: What are logarithms and what do their graphs look like? What is the difference between a logarithm and a natural logarithm?	**Overarching Enduring Understandings:** Functions include exponential, logarithmic, and those with domains and ranges that are limited and/or discontinuous. Exponential and logarithmic functions are either strictly increasing or strictly decreasing.
Unit Essential Questions: What is the relationship between logarithms and exponential equations? How can they be identified from their graphs? How can an exponential equation be constructed to solve a real world problem? How can operations be extended over exponential, logarithms, natural log and natural base? How can equations involving logarithms, natural log and natural base be solved?	**Unit Enduring Understandings:** Logarithms and exponents are inverse notations. Logs are used to solve problems involving unknown exponent values and are still relied upon heavily in business, science, and financial formulas.
Benchmarks: Algebra 2 Holt/ McDougal- Assessment Book: Ch 3 Assessment Prentice Hall- Algebra 2 Progress Monitoring Assessments- Benchmark Tests.	**Unit Student Learning Overview:** Students will be able to use exponential functions to model changes in the values of dependent variables. Students will also be able to develop properties of logarithms and use the properties to solve problems algebraically.

Key Terms (Essential Vocabulary):
Discontinuous- not connected
Exponential- having an exponent
Logarithm- expression converted out of exponential form

Resources:
Algebra 2 (ISBN 0-03-066054-8)
TI 83 or 84 Graphing Calculator
TI Exploration Activities – see TI web site for up to date plans and worksheets
Worksheets and warm up from www.kuta.com
Videos from www.brightstorm.com and www.khanacademy.com
Desmos www.desmos.com
Three Acts
https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTQEsfqouLWNNC6Z4/edit#gid=0

Suggested Activities for Inclusion in Lesson Planning
Interdisciplinary Connections are identified with and I, followed by the related content area(s):
Predict value of investment for different amounts and different lengths of time.
Use logs to find out rates of investment.
The focus of this unit should be the compound interest formulas and the conversion from log to exponential to
solve science application problems.
Investigate the impact of compounding interest by calculating the value of several investments over time and with varying compounding intervals. Students will calculate and graph on graph paper, the value of an account with a set interest rate over a period of 10 years, then the same account with compounding

semi- annually, quarterly, and monthly, over the same 10 year period graph simultaneously. Students will compare and contrast the impact of the changing various elements of the formula.
A single-celled bacterium divides every hour. The number N of bacteria after t hours is given by the formula $\log_2 N=t$. a) After how many hours will there be 64 bacterium? b) Explain in words or show work for how you determined the number of hours.
Given the graphs, recognize that exponential and logarithmic functions are inverses of each other.
Investigate exponential and logarithmic functions, using the graphing calculator.
Investigate the shape and behavior of exponential (ax = y) and logarithmic (log b x = y) functions, including intercepts and end behavior.
Determine the multiplier for growth and decay then write, classify and evaluate exponential equations to model them.
Solve, simplify and evaluate expressions and equations involving logarithms, natural log and natural base.
Solve and model real world problems involving exponential equations, logarithms, natural log and natural base.

Modifications for Special Education Students, ELLs and Gifted Students:	**Suggested Timeline:**
Special Needs – Students with IEPs will be placed in classes with additional instructional support, and the material will be delivered in a co-teaching model. Students with 504s will receive the support those documents dictate. **ELL** – Language support, as needed. Utilization of experience and information, as applicable. Opportunities for students to write or communicate in their native language, as availability of translation allows. Depending on the level of acquisition, opportunities to write instead of speaking; and opportunities to practice speaking.	Logarithmic and Exponential Expressions and Equations: **1-2 weeks** Construct and Analyze Exponential Functions, Interpret Exponential and Logarithmic Functions: **1-2 weeks**

Gifted Learners – Deeper investigations of content; lead roles during collaborative group assignments. **Mainstream Learners** – Formative assessments to gauge understanding and learning; participation in lesson examples.	

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Written By: Ron Latham, Patricia Martel
Course Title: Algebra II Unit Name: Probability & Statistics
Grade Level: 9-12

Content Statements and Rationale:	**NJSLS:**
In this unit, students will strengthen their understanding of simple and compound events, and building on those concepts, as well as the	S.CP.1-9 S.IC.1- 6 S.ID.4

fundamental counting principles and notion of independence.	S.MD.6-7
Overarching Essential Questions: How can statistics be used to solve problems? How does position or arrangement shape impact probability?	**Overarching Enduring Understandings:** Data and scatter plots may indicate patterns that can be modeled with an algebraic equation. Graphing calculators can be used to collect, organize, picture, and create an algebraic model of the data. Data that fit linear, quadratic, exponential, and logarithmic models arise from practical situations.
Unit Essential Questions: What graphic organizer best represents different types of data? How can scatter plots be used to find patterns? How can the equation of best fit for data be found? How can graphic organizers be used to make predictions? How do you find the probability when order is involved? When is order important? What changes when items are arranged in a circle? What changes when items are repeated?	**Unit Enduring Understandings:** The desired results can help choose the best graphic organizer for a given set of data. Correlations and lines of best fit can be used to find patterns in scatterplots and to make predictions.
Benchmarks: Algebra 2 Holt/ McDougal- Assessment Book: Ch 8-9 Assessment Prentice Hall- Algebra 2 Progress Monitoring Assessments- Benchmark Tests.	**Unit Student Learning Overview:** Students will be able to use probability, relative frequencies, and discrete distributions to develop a conceptual understanding of the normal distribution and use the distribution to estimate population proportions.

Key Terms (Essential Vocabulary):
Best Fit Line- using regression and scatterplots
Combination- an arrangement of objects where the order is not important
Permutation- an arrangement of objects where the order is importation
Outlier- data value far apart from the rest that skews the overall data

Resources:
Algebra 2 (ISBN 0-03-066054-8)
TI 83 or 84 Graphing Calculator
TI Exploration Activities – see TI web site for up to date plans and worksheets
Worksheets and warm up from www.kuta.com
Videos from www.brightstorm.com and www.khanacademy.com
Desmos www.desmos.com
Three Acts
https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTQEsfqouLWNNC6Z4/edit#gid=0

Suggested Activities for Inclusion in Lesson Planning
Interdisciplinary Connections are identified with and I, followed by the related content area(s):
Complete experiments involving probability. Test theoretical calculations with physical trials.
Collect and analyze data.
Investigate scatter-plots to determine if patterns exist, and then identify the patterns. Collect result data from the 2014 Summer Games and calculating the mean deviation, standard deviation, creating box whisker plots, and scatterplots for student selected medal events.
Use real world problems to solve for probability and statistics
Find an equation for the curve of best fit for data, using a graphing calculator. Models will include linear, quadratic, exponential, and logarithmic functions.
Make predictions, using data, scatterplots, or curve of best fit.
Given a set of data, determine the model that would best describe the data.

Modifications for Special Education Students, ELLs and Gifted Students:	**Suggested Timeline:**

Special Needs – Students with IEPs will be placed in classes with additional instructional support, and the material will be delivered in a co-teaching model. Students with 504s will receive the support those documents dictate. **ELL** – Language support, as needed. Utilization of experience and information, as applicable. Opportunities for students to write or communicate in their native language, as availability of translation allows. Depending on the level of acquisition, opportunities to write instead of speaking; and opportunities to practice speaking. **Gifted Learners** – Deeper investigations of content; lead roles during collaborative group assignments. **Mainstream Learners** – Formative assessments to gauge understanding and learning; participation in lesson examples.	Rules of Probability to Compute Compound Events, Using Probability to Make Decisions: **1 week** The Design of Statistical Studies: **1 week** Gathering Data, Making Inferences, and Justifying Conclusions: **1 week**

Content Statements and Rationale: In this unit, students are introduced to periodic functions and three trigonometric functions: $y = \sin x$, $y = \cos x$, and $y = \tan x$. Students will learn to transform these functions just as they have transformed linear, quadratic, and exponential functions.	**NJSLS:** N.Q.2 F.IF.4, 6, 7, 9 F.BF.3 F.TF.1-2, 5, 8
Overarching Essential Questions: How can trigonometric properties be used to solve a right triangle? How can triangles relate to a circle? How can trigonometry be applied to any type of triangle, not just right triangles? How are trigonometric functions related to anything in the real world? How can the Law of Sines and Law of Cosines be used to solve non-right triangles?	**Overarching Enduring Understandings:** Using the three basic trigonometric functions and a right triangle, we can compare the opposite side, adjacent side, and hypotenuse to find missing parts of a triangle. Trigonometric functions can be used to model real-world problems, including right triangle relations, arc length, and speed.
Unit Essential Questions: What is right triangle trigonometry? How can right triangle definitions be extended to apply to circular functions? What are the three basic trigonometric functions? How is algebra used in trigonometry? How can we use these functions to solve right triangles?	**Unit Enduring Understandings:** Trigonometry can be used not just to solve triangles, but to model waves. The important properties of waves, including amplitude, period, and shift, can be found by applying algebra to the trigonometric function.
Benchmarks: Algebra 2 Holt/ McDougal- Assessment Book: Ch 10 Assessment	**Unit Student Learning Overview:** Students will be able to understand how trigonometric functions are generated and used to model various situations.

Prentice Hall- Algebra 2 Progress Monitoring Assessments- Benchmark Tests.	

Key Terms (Essential Vocabulary):

Sine, Cosine, Tangent- three basic trigonometric functions used in right triangle trigonometry

Degrees and Radian- measurements of an angle

Amplitude- the difference between the median and its extrema in sine and cosine, change in amplitude in music changes volume

Frequency- the number of periods occurring within the standard period, change in frequency in sound waves determines pitch

Inverse or Arc- the function that is applied to undo or peal back the given trig function, a method of solution

Period- length of piece being repeated to generate trig graph

Resources:

Algebra 2 (ISBN 0-03-066054-8)

TI 83 or 84 Graphing Calculator

TI Exploration Activities – see TI web site for up to date plans and worksheets

Worksheets and warm up from www.kuta.com

Videos from www.brightstorm.com and www.khanacademy.com

Desmos www.desmos.com

Three Acts

https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTQEsfqouLWNNC6Z4/edit#gid=0

Suggested Activities for Inclusion in Lesson Planning
Interdisciplinary Connections are identified with and I, followed by the related content area(s):
Solving right triangles
Creating reference angles to convert circular functions into right triangles
Sketching the unit circle and entering values
Finding values of circular functions on the unit circle
Applying the law of sines and cosines to non-right triangles
Determining whether an SSA case describes 0, 1 or 2 triangles

Modifications for Special Education Students, ELLs and Gifted Students:

Special Needs – Students with IEPs will be placed in classes with additional instructional support, and the material will be delivered in a co-teaching model. Students with 504s will receive the support those documents dictate.

ELL – Language support, as needed. Utilization of experience and information, as applicable. Opportunities for students to write or communicate in their native language, as availability of translation allows. Depending on the level of acquisition, opportunities to write instead of speaking; and opportunities to practice speaking.

Gifted Learners – Deeper investigations of content; lead roles during collaborative group assignments.

Mainstream Learners – Formative assessments to gauge understanding and learning; participation in lesson examples.

Suggested Timeline:
Interpreting and Analyzing Trigonometric Functions: **1 week**

Unit Circle: **1 week**

Identifying, Applying and Proving Trigonometric Identities: **1 week**

Appendix

Differentiation	
Enrichment	● Utilize collaborative media tools ● Provide differentiated feedback ● Opportunities for reflection ● Encourage student voice and input ● Model close reading ● Distinguish long term and short term goals
Intervention & Modification	● Utilize “skeleton notes” where some required information is already filled in for the student ● Provide access to a variety of tools for responses ● Provide opportunities to build familiarity and to practice with multiple media tools ● Leveled text and activities that adapt as students build skills ● Provide multiple means of action and expression ● Consider learning styles and interests ● Provide differentiated mentors ● Graphic organizers

ELLs	• Pre-teach new vocabulary and meaning of symbols • Embed glossaries or definitions • Provide translations • Connect new vocabulary to background knowledge • Provide flash cards • Incorporate as many learning senses as possible • Portray structure, relationships, and associations through concept webs • Graphic organizers

21st Century Skills

- Creativity
- Innovation
- Critical Thinking
- Problem Solving
- Communication
- Collaboration

Integrating Technology

- Chromebooks
- Internet research
- Online programs
- Virtual collaboration and projects
- Presentations using presentation hardware and software